# 4.8 Binomial distribution\_P\_2

**1a.** *[3 marks]*

A café serves sandwiches and cakes. Each customer will choose one of the following three options; buy only a sandwich, buy only a cake or buy both a sandwich and a cake.

The probability that a customer buys a sandwich is 0.72 and the probability that a customer buys a cake is 0.45.

Find the probability that a customer chosen at random will buy

both a sandwich and a cake.

**1b.** *[1 mark]*

only a sandwich.

**1c.** *[1 mark]*

On a typical day 200 customers come to the café.

Find the expected number of cakes sold on a typical day.

**1d.** *[3 marks]*

Find the probability that more than 100 cakes will be sold on a typical day.

**1e.** *[1 mark]*

It is known that 46 % of the customers who come to the café are male, and that 80 % of these buy a sandwich.

A customer is selected at random. Find the probability that the customer is male and buys a sandwich.

**1f.** *[4 marks]*

A female customer is selected at random. Find the probability that she buys a sandwich.

**2a.** *[2 marks]*

Steffi the stray cat often visits Will’s house in search of food. Let  be the discrete random variable “the number of times per day that Steffi visits Will’s house”.

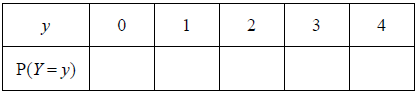
The random variable  can be modelled by a Poisson distribution with mean 2.1.

Find the probability that on a randomly selected day, Steffi does not visit Will’s house.

**2b.** *[4 marks]*

Let *Y* be the discrete random variable “the number of times per day that Steffi is fed at Will’s house”. Steffi is only fed on the first four occasions that she visits each day.

Copy and complete the probability distribution table for *Y*.



**2c.** *[3 marks]*

Hence find the expected number of times per day that Steffi is fed at Will’s house.

**2d.** *[3 marks]*

In any given year of 365 days, the probability that Steffi does not visit Will for at most  days in total is 0.5 (to one decimal place). Find the value of .

**2e.** *[4 marks]*

Show that the expected number of occasions per year on which Steffi visits Will’s house and is not fed is at least 30.

**3a.** *[2 marks]*

Timmy owns a shop. His daily income from selling his goods can be modelled as a normal distribution, with a mean daily income of 230. To make a profit, Timmy’s daily income needs to be greater than $1000.

Calculate the probability that, on a randomly selected day, Timmy makes a profit.

**3b.** *[3 marks]*

The shop is open for 24 days every month.

Calculate the probability that, in a randomly selected month, Timmy makes a profit on between 5 and 10 days (inclusive).

**4a.** *[2 marks]*

Willow finds that she receives approximately 70 emails per working day.

She decides to model the number of emails received per working day using the random variable , where  follows a Poisson distribution with mean 70.

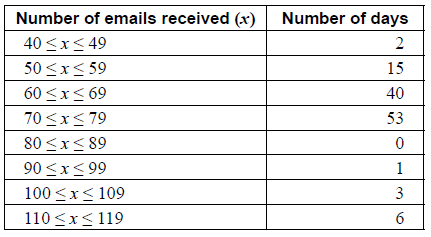
Using this distribution model, find .

**4b.** *[2 marks]*

Using this distribution model, find the standard deviation of .

**4c.** *[3 marks]*

In order to test her model, Willow records the number of emails she receives per working day over a period of 6 months. The results are shown in the following table.



From the table, calculate

an estimate for the mean number of emails received per working day.

**4d.** *[2 marks]*

an estimate for the standard deviation of the number of emails received per working day.

**4e.** *[1 mark]*

Give one piece of evidence that suggests Willow’s Poisson distribution model is not a good fit.

**4f.** *[3 marks]*

Archie works for a different company and knows that he receives emails according to a Poisson distribution, with a mean of  emails per day.

Suppose that the probability of Archie receiving more than 10 emails in total on any one day is 0.99. Find the value of *λ*.

**4g.** *[5 marks]*

Now suppose that Archie received exactly 20 emails in total in a consecutive two day period. Show that the probability that he received exactly 10 of them on the first day is independent of *λ*.

**5.** *[5 marks]*

The mean number of squirrels in a certain area is known to be 3.2 squirrels per hectare of woodland. Within this area, there is a 56 hectare woodland nature reserve. It is known that there are currently at least 168 squirrels in this reserve.

Assuming the population of squirrels follow a Poisson distribution, calculate the probability that there are more than 190 squirrels in the reserve.

**6a.** *[2 marks]*

The random variable *X* has a binomial distribution with parameters *n* and *p*.  
It is given that E(*X*) = 3.5.

Find the least possible value of *n*.

**6b.** *[5 marks]*

It is further given that P(*X* ≤ 1) = 0.09478 correct to 4 significant figures.

Determine the value of *n* and the value of *p*.

**7a.** *[2 marks]*

The number of taxis arriving at Cardiff Central railway station can be modelled by a Poisson distribution. During busy periods of the day, taxis arrive at a mean rate of 5.3 taxis every 10 minutes. Let T represent a random 10 minute busy period.

Find the probability that exactly 4 taxis arrive during T.

**7b.** *[2 marks]*

Find the most likely number of taxis that would arrive during T.

**7c.** *[3 marks]*

Given that more than 5 taxis arrive during T, find the probability that exactly 7 taxis arrive during T.

**7d.** *[6 marks]*

During quiet periods of the day, taxis arrive at a mean rate of 1.3 taxis every 10 minutes.

Find the probability that during a period of 15 minutes, of which the first 10 minutes is busy and the next 5 minutes is quiet, that exactly 2 taxis arrive.

**8a.** *[2 marks]*

The number of bananas that Lucca eats during any particular day follows a Poisson distribution with mean 0.2.

Find the probability that Lucca eats at least one banana in a particular day.

**8b.** *[4 marks]*

Find the expected number of weeks in the year in which Lucca eats no bananas.

**9a.** *[3 marks]*

When carpet is manufactured, small faults occur at random. The number of faults in Premium carpets can be modelled by a Poisson distribution with mean 0.5 faults per 20m2. Mr Jones chooses Premium carpets to replace the carpets in his office building. The office building has 10 rooms, each with the area of 80m2.

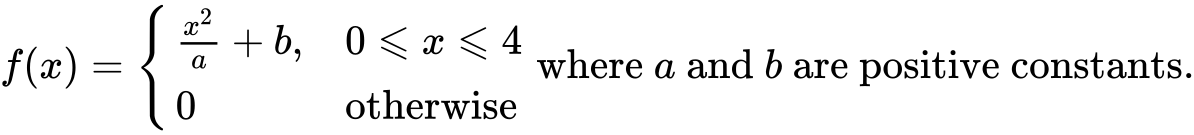
Find the probability that the carpet laid in the first room has fewer than three faults.

**9b.** *[3 marks]*

Find the probability that exactly seven rooms will have fewer than three faults in the carpet.

**10a.** *[5 marks]*

A continuous random variable  has probability density function  given by



It is given that .

Show that  and .

**10b.** *[2 marks]*

Find .

**10c.** *[2 marks]*

Find .

**10d.** *[3 marks]*

Find the median of .

**10e.** *[2 marks]*

Eight independent observations of  are now taken and the random variable  is the number of observations such that .

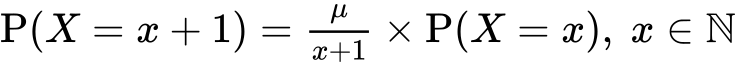
Find .

**10f.** *[1 mark]*

Find .

**11a.** *[3 marks]*

A discrete random variable  follows a Poisson distribution .

Show that .

**11b.** *[3 marks]*

Given that  and , use part (a) to find the value of .

**12.** *[8 marks]*

Students sign up at a desk for an activity during the course of an afternoon. The arrival of each student is independent of the arrival of any other student and the number of students arriving per hour can be modelled as a Poisson distribution with a mean of .

The desk is open for 4 hours. If exactly 5 people arrive to sign up for the activity during that time find the probability that exactly 3 of them arrived during the first hour.

**13a.** *[2 marks]*

Six balls numbered 1, 2, 2, 3, 3, 3 are placed in a bag. Balls are taken one at a time from the bag at random and the number noted. Throughout the question a ball is always replaced before the next ball is taken.

A single ball is taken from the bag. Let  denote the value shown on the ball.

Find .

**13b.** *[3 marks]*

Three balls are taken from the bag. Find the probability that

the total of the three numbers is 5;

**13c.** *[3 marks]*

the median of the three numbers is 1.

**13d.** *[3 marks]*

Ten balls are taken from the bag. Find the probability that less than four of the balls are numbered 2.

**13e.** *[3 marks]*

Find the least number of balls that must be taken from the bag for the probability of taking out at least one ball numbered 2 to be greater than 0.95.

**13f.** *[8 marks]*

Another bag also contains balls numbered 1 , 2 or 3.

Eight balls are to be taken from this bag at random. It is calculated that the expected number of balls numbered 1 is 4.8 , and the variance of the number of balls numbered 2 is 1.5.

Find the least possible number of balls numbered 3 in this bag.

**14a.** *[3 marks]*

A company produces rectangular sheets of glass of area 5 square metres. During manufacturing these glass sheets flaws occur at the rate of 0.5 per 5 square metres. It is assumed that the number of flaws per glass sheet follows a Poisson distribution.

Find the probability that a randomly chosen glass sheet contains at least one flaw.

**14b.** *[3 marks]*

Glass sheets with no flaws earn a profit of 3.

Find the expected profit,  dollars, per glass sheet.

**14c.** *[2 marks]*

This company also produces larger glass sheets of area 20 square metres. The rate of occurrence of flaws remains at 0.5 per 5 square metres.

A larger glass sheet is chosen at random.

Find the probability that it contains no flaws.

**15a.** *[6 marks]*

A survey is conducted in a large office building. It is found that  of the office workers weigh less than  kg and that  of the office workers weigh more than  kg.

The weights of the office workers may be modelled by a normal distribution with mean  and standard deviation .

(i)     Determine two simultaneous linear equations satisfied by  and .

(ii)     Find the values of  and .

**15b.** *[1 mark]*

Find the probability that an office worker weighs more than  kg.

**15c.** *[2 marks]*

There are elevators in the office building that take the office workers to their offices.

Given that there are  workers in a particular elevator,

find the probability that at least four of the workers weigh more than  kg.

**15d.** *[3 marks]*

Given that there are  workers in an elevator and at least one weighs more than  kg,

find the probability that there are fewer than four workers exceeding  kg.

**15e.** *[3 marks]*

The arrival of the elevators at the ground floor between  and  can be modelled by a Poisson distribution. Elevators arrive on average every  seconds.

Find the probability that in any half hour period between  and  more than  elevators arrive at the ground floor.

**15f.** *[3 marks]*

An elevator can take a maximum of  workers. Given that  workers arrive in a half hour period independently of each other,

find the probability that there are sufficient elevators to take them to their offices.

**16a.** *[2 marks]*

A mosaic is going to be created by randomly selecting 1000 small tiles, each of which is either black or white. The probability that a tile is white is 0.1. Let the random variable  be the number of white tiles.

State the distribution of , including the values of any parameters.

**16b.** *[1 mark]*

Write down the mean of .

**16c.** *[2 marks]*

Find .

**17a.** *[3 marks]*

The random variable  follows a Poisson distribution with mean .

Given that , show that .

**17b.** *[4 marks]*

Given that , find the probability that .

**18a.** *[6 marks]*

Farmer Suzie grows turnips and the weights of her turnips are normally distributed with a mean of  and standard deviation of .

(i)     Calculate the percentage of Suzie’s turnips that weigh between  and .

(ii)     Suzie has  turnips to take to market. Find the expected number weighing more than .

(iii)     Find the probability that at least  of the  turnips weigh more than .

**18b.** *[6 marks]*

Farmer Ray also grows turnips and the weights of his turnips are normally distributed with a mean of . Ray only takes to market turnips that weigh more than . Over a period of time, Ray finds he has to reject  in  turnips due to their being underweight.

(i)     Find the standard deviation of the weights of Ray’s turnips.

(ii)     Ray has  turnips to take to market. Find the expected number weighing more than .

**19a.** *[4 marks]*

Emma acquires a new cell phone for her birthday and receives texts from her friends. It is assumed that the daily number of texts Emma receives follows a Poisson distribution with mean .

(i)     Find the probability that on a certain day Emma receives more than  texts.

(ii)     Determine the expected number of days in a week on which Emma receives more than  texts.

**19b.** *[3 marks]*

Find the probability that Emma receives fewer than  texts during a week.

**20a.** *[3 marks]*

The number of complaints per day received by customer service at a department store follows a Poisson distribution with a mean of .

On a randomly chosen day, find the probability that

(i)     there are no complaints;

(ii)     there are at least three complaints.

**20b.** *[2 marks]*

In a randomly chosen five-day week, find the probability that there are no complaints.

**20c.** *[3 marks]*

On a randomly chosen day, find the most likely number of complaints received.

Justify your answer.

**20d.** *[2 marks]*

The department store introduces a new policy to improve customer service. The number of complaints received per day now follows a Poisson distribution with mean .

On a randomly chosen day, the probability that there are no complaints is now .

Find the value of .

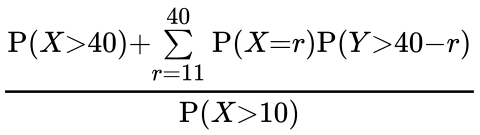
**21a.** *[2 marks]*

The number of birds seen on a power line on any day can be modelled by a Poisson distribution with mean 5.84.

Find the probability that during a certain seven-day week, more than 40 birds have been seen on the power line.

**21b.** *[5 marks]*

On Monday there were more than 10 birds seen on the power line. Show that the probability of there being more than 40 birds seen on the power line from that Monday to the following Sunday, inclusive, can be expressed as:

 where  and .

**22.** *[7 marks]*

Six customers wait in a queue in a supermarket. A customer can choose to pay with cash or a credit card. Assume that whether or not a customer pays with a credit card is independent of any other customers’ methods of payment.

It is known that 60% of customers choose to pay with a credit card.

(a)     Find the probability that:

          (i)     the first three customers pay with a credit card and the next three pay with cash;

          (ii)     exactly three of the six customers pay with a credit card.

There are *n* customers waiting in another queue in the same supermarket. The probability that at least one customer pays with cash is greater than 0.995.

(b)     Find the minimum value of *n*.

**23.** *[4 marks]*

The random variable  has a Poisson distribution with mean .

Given that ,

(a)     find the value of ;

(b)     find the probability that *X* lies within one standard deviation of the mean.

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